Logic of Defeasible Permission and its Dynamics

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Defeasible Permission and Normative Conflicts

You can take off your heart-rate monitor when exercising at school.
当在学校运动时你可以摘下心率监测仪。

You cannot take off your heart-rate monitor.
你不能摘下心率监测仪。

progressive relationship: connected by ``and'' 并且

You can take part in physical exercise at school.
你可以参加学校的体育活动。

adversative relationship: connected by ``but'' 但是

progressive relationship: connected by ``and'' 并且

You can go to school.
你可以上学。

When your doctor tells you that you are recovered well, you can take the monitor off.
当你的医生告诉你已经完全恢复时，你可以摘下监测仪。

A scenario of recording the heart-rate of Ann who is risking heart-attack (in Chinese language)

Research Questions

1. What is the logical structure to infer another permission from the given one, without normative violation.
2. When a permission can be preserved, violated, or added?

Outlines

- A Brief History of Permission
- A Modal Logic of Defeasible Permission
- Two Dynamics of Updating Permissions

Permission as Deontic Sufficiency

- The Semantics of (Free Choice/Strong) Permission

(von Wright 1951, Dignum et al. 1996, van Benthem 1979)

- φ is permitted ($P\varphi$) when its performance never lead to violation
-

$$M, w \models P\varphi \text{ iff } ||\varphi|| \subseteq I[w]$$

where ||φ|| is the set of worlds make φ true.

- Free Choice Permission (FCP)
- Monotonic Reasoning:

$$P(\varphi \vee \psi) \rightarrow P\varphi \wedge P\psi$$

$$P\varphi \rightarrow P(\varphi \wedge \psi)$$

$$O\neg\psi, P\varphi \models P(\varphi \wedge \psi), O\neg\psi$$

Defeasible Permission

- Permission as Normic Laws (Lewis 1979, Pelletier and Asher 1997)
- φ is permitted when its performance, normally, does not lead to violation

- The Inferences of Defeasible Permission:
- (Normally) You can go to school.
- (Normally) You can take part in physical exercise at school.
- (Normally) You can take off your heart-rate monitor when exercising at school.

We need to model Defeasibility and Ideality at the same time.

$$\mathrm{Best}(||\varphi||) \subseteq I[w]$$

Preference-based Deontic Logic

- Preference as Normality in Deontic Logic:

  (van Fraassen 1973), (Hansson 1970)

- (Multi-) Preference-based Deontic Logic for (Dyadic) Obligation:

  O(ψ/φ) is true at model w iff "ψ is preferable than ¬ψ, conditional on the most normal φ-worlds"

  (Boutilier 1994-KR), (Halpern 1997), (van der Torre 1997)

and its dynamics (Veltman 1996, Lang et al. 2003, Lang et al. 2008)

Modal Logic of Defeasible Permission

- Deontic Language: $\varphi := p \mid \varphi \wedge \varphi \mid \neg\varphi \mid P\varphi \mid O\varphi \mid \varphi \trianglelefteq \varphi$

- Deontic Model M = $\langle W, I, \leq, V \rangle$ where:

① W is non-empty;

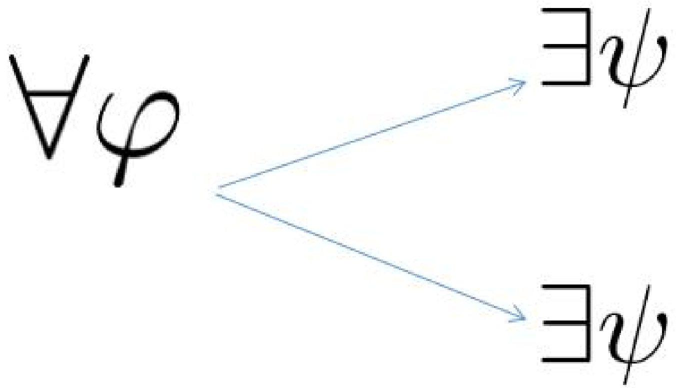
② Ideality $I \subseteq W \times W$ is serial and the likelihood ≤ is reflexive and transitive;

③ V is a valuation function.

Truth Conditions of Normality

- Binary modality $\trianglelefteq$: "$\varphi \trianglelefteq \psi$" is read as "φ is more likely than ψ"
- First proposed by von Wright (1963), later discussed by Boutilier (1994, KR), Halpern (1997), van der Torre (1997), and van Benthem et al. (2009).

$$M, w \models \varphi \trianglelefteq \psi \text{ iff } \forall u \in W \exists v \in W \text{ s.t. } (M, u \models \varphi \Rightarrow M, v \models \psi \ \& \ u \leq v)$$



Truth Conditions of Normality

- "Most normal" is defined as the maximality of likelihood, as suggested by Burgess (1981):

$$M, w \models \Box(\psi/\varphi) \text{ iff } \forall v \in W\ [M, v \models \varphi \,\&\, \exists u \geq v\ (M, u \models \varphi\ \&\ \forall s \geq u\ (M, s \models \varphi \Rightarrow M, s \models \psi))]$$

- $\Box(\psi/\varphi)$: "φ is a normal instance of ψ" when every most likely φ-case is a ψ-case.
- The interplay between likelihood and the maximality is (Halpern 1997):

$$\Box(\psi/\varphi) \leftrightarrow [E\varphi \rightarrow (\varphi \wedge \neg\psi) \trianglelefteq (\varphi \wedge \psi)]$$

$$\forall\varphi \rightarrow \exists\varphi \longrightarrow ||\varphi|| \subseteq ||\psi||$$

Truth Conditions of Defeasible Permission

- Obligation is defined as in Standard Deontic Logic.
- A permission holds when "every most likely case is ideal":

$$M, w \models O\varphi \text{ iff } R[w] \subseteq ||\varphi||$$
$$M, w \models P\varphi \text{ iff } \forall v \in W\ [M, v \models \varphi \Rightarrow \exists u \geq v\ (M, u \models \varphi\ \&\ \forall s \geq u\ (M, s \models \varphi \Rightarrow s \in I[w]))]$$

- "Every most likely case of doing exercise at school is ideal."

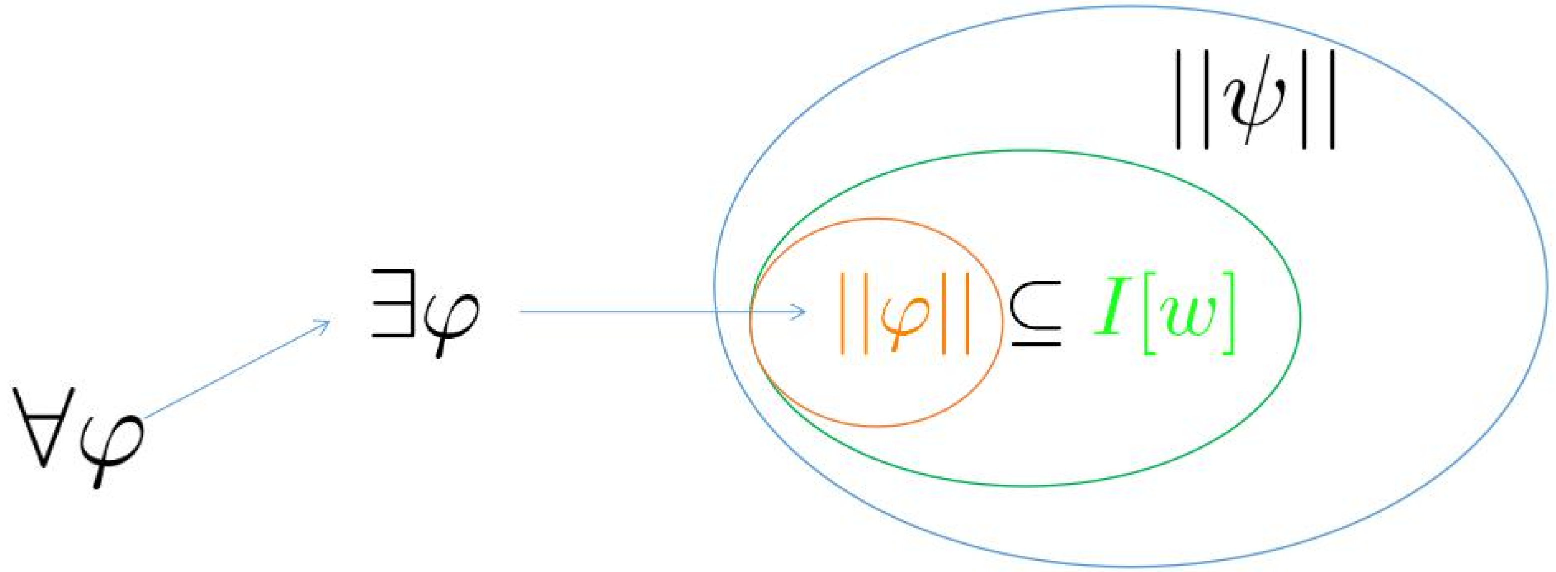
$$\forall\varphi \rightarrow \exists\varphi \rightarrow ||\varphi|| \subseteq I[w]$$

To bound defeasible permission

- Every permitted action is a normal instance of an obligatory action:

$$O\varphi \wedge P\psi \to \Box(\varphi/\psi)$$

- "For Ann execising at school is a normal instance of not taking off the heart-rate monitor."



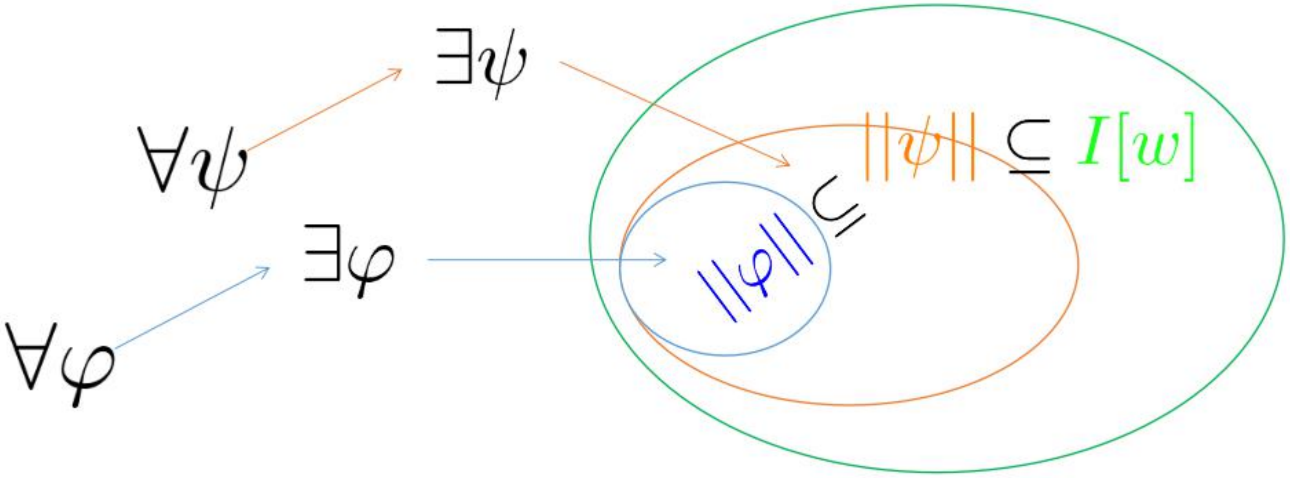
To imply defeasible permission

- When an action of permitted, every proper instance of it is also permitted:

$$P\psi \wedge \heartsuit(\psi/\varphi) \rightarrow P\varphi$$

where $\heartsuit(\psi/\varphi)$ indicates "φ is a proper instance of ψ", "every most likely φ-case is also a most likely ψ-case" equal to $(\psi \trianglelefteq \varphi) \wedge (\varphi \trianglelefteq \psi) \wedge \Box(\psi/\varphi)$

- "For Ann NOT taking off the monitor when exercising is a proper instance of exercising at school".



Axiomatization

- Tautologies
- The binary modality $\trianglelefteq$ satisfies:

  REF: $A(\psi \to \varphi) \to \psi \trianglelefteq \varphi$

  TRAN: $(\varphi \trianglelefteq \chi) \wedge E(\chi \trianglelefteq \psi) \to (\varphi \trianglelefteq \psi)$

  $\bot_{\trianglelefteq:l}$: $\bot \trianglelefteq \varphi$

  $\bot_{\trianglelefteq:r}$: $\neg(\varphi \trianglelefteq \bot)$ where $\varphi \neq \bot$

  $\mathrm{RM}_{\trianglelefteq:l}$: $\chi \to \varphi \vee \psi / \varphi \trianglelefteq \delta \wedge \psi \trianglelefteq \delta \to \chi \trianglelefteq \delta$

  $\mathrm{RM}_{\trianglelefteq:r}$: $\varphi \to \psi / \chi \trianglelefteq \varphi \to \chi \trianglelefteq \psi$

  CM: $\Box(\psi/\varphi) \wedge \Box(\chi/\varphi) \to \Box(\chi/\varphi \wedge \psi)$

- The reduction axioms:

  $\mathrm{Red}_{\Box}$: $\Box(\psi/\varphi) \leftrightarrow [E\varphi \to (\varphi \wedge \neg\psi) \trianglelefteq (\varphi \wedge \psi)]$

  $\mathrm{Red}_{\heartsuit}$: $\heartsuit(\psi/\varphi) \leftrightarrow (\psi \trianglelefteq \varphi) \wedge (\varphi \trianglelefteq \psi) \wedge \Box(\psi/\varphi)$

- The modalities A and E are defined as:

  $A\varphi \leftrightarrow (\neg\varphi) \trianglelefteq \bot$

  $E\varphi \leftrightarrow \neg A \neg\varphi$

- The modality P satisfies:

  PDS: $O\varphi \wedge P\psi \to \Box(\varphi/\psi)$

  PtF: $P\bot$

  FCP: $P\psi \wedge \heartsuit(\psi/\varphi) \to P\varphi$

  FCC: $P\varphi \wedge P\psi \to P(\varphi \vee \psi)$

The modality O satisfies:

  K_O: $O(\varphi \to \psi) \wedge O\varphi \to O\psi$

  NEC_O: $\varphi / O\varphi$

  D_O: $\neg O\bot$

This logic is sound and (strong) complete.

Two Dynamics

- The view of Lewis (1979):

① Permission can be eliminated by introducing a prohibition;

② Permission can be added by extending the space of permissibility.

(Hanson 2013)

- The updates from changing likelihood to changing norms (Lang et al. 2003, 2008).
- When one norm has more specific reason, it can be used to defeat its contrary (by DEL models).

Ranking Models for Specificity

- Define a ranking model $C^{\Gamma} = \langle C(\Gamma), \preceq \rangle$ to represent the specific statuses in a context Γ :
- $$C(\Gamma) = \{\{\pm p \mid p \text{ is an atomic proposition occurs in } \Gamma\} \mid \text{either } \pm p = p \text{ or } \pm p = \neg p\};$$
- $\preceq \subseteq C \times C$ is reflexive and transitive.

- (Hansson 1990, Wellman et al. 1991, Girard et al. 2017)

Contraction Update

Definition 5.2 (Contraction Update). Given $M = \langle W, R, \leq, V \rangle$ and $C^{\Gamma} = \langle C, \preceq \rangle$. We define the contraction-updated model $M \otimes C^{\Gamma} = \langle W^*, R^*, \leq^*, V^* \rangle$ as follows:

- $W^* = \{(u, c) \mid M, u \vDash c \text{ where } c \in C\}$;
- $(u, c) \leq^* (v, d)$ iff
  - either $c \prec d$
  - or $(c \sim d$ and $u \leq v)$;
- $(u, c) R^* (v, d)$ iff uRv and $c \preceq d$, such that it is serial:

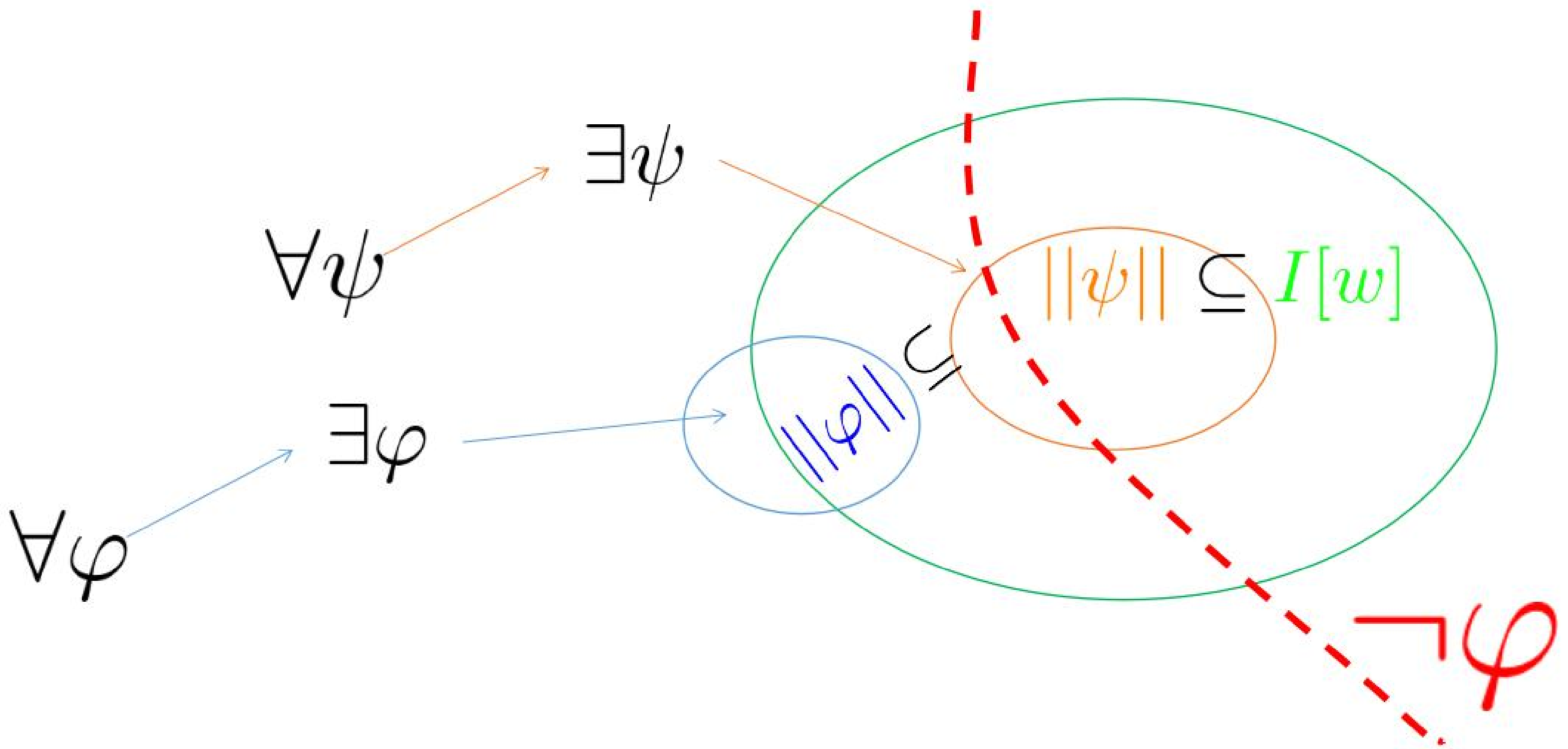
$$\forall (u, c) \exists (v, d) \text{ s.t. } (c \preceq d \,\&\, uRv);$$

- $(u, c) \in V^*(p)$ iff $u \in V(p)$.

The necessary additional axiom for this updated model is: $c \rightarrow \bigvee_{d \succeq c} \neg O \neg d$.

“NOT take off the monitor!”

- φ is “Take off your heart-rate monitor”
- ψ is “Take part in physical exercise at school”



$$M, w \models \langle \Gamma \rangle \varphi \text{ iff } \exists (w, c) \in W^* \text{ s.t. } M \otimes C^{\Gamma}, (w, c) \models \varphi$$

Reduction Axioms of Contraction Update

- $[\Gamma]p \leftrightarrow \bigwedge_{c \in C}(\bigwedge c \rightarrow p)$
- $[\Gamma]\varphi \wedge \psi \leftrightarrow [\Gamma]\varphi \wedge [\Gamma]\psi$
- $[\Gamma]\neg\varphi \leftrightarrow \bigwedge_{c \in C}(\bigwedge c \rightarrow \neg[\Gamma]\varphi)$
- $[\Gamma]O\varphi \leftrightarrow \bigwedge_{c \in C}[c \rightarrow \bigwedge_{d \geq c} O\varphi^{d}]$
- $[\Gamma]P\varphi \leftrightarrow [(\bigwedge |\varphi| \leftrightarrow \bot) \rightarrow \bigwedge_{c \in C}[c \rightarrow \bigwedge_{d \in \max(|\varphi|) \cap \geq [c]} P \bigvee_{b \sim d} \varphi^{b}]]$
- $[\Gamma](\varphi \trianglelefteq \psi) \leftrightarrow \bigwedge_{c \in C} \bigwedge_{d \in C}[c \rightarrow \bigvee_{e \succ d} A(\varphi^{d} \rightarrow E\psi^{e}) \vee \bigvee_{e \sim d}(\varphi^{d} \trianglelefteq \psi^{e})]$

Lexicographic Update

Definition 5.3 (Lexicographic Update). Given $M = \langle W, R, \leq, V\rangle$ and $C^\Gamma = \langle C, \preceq\rangle$. We define the lexicographic-updated model $M \otimes C^\Gamma = \langle W^*, R^*, \leq^*, V^*\rangle$ as follows:

- $W^* = \{(u, c) \mid M, u \vDash c \text{ where } c \in C\}$; [14]
- $(u, c) \leq^* (v, d)$ iff
  - either $c \prec d$
  - or ($c \sim d$ and $u \leq v$);
- $(u, c)R^*(v, d)$ iff
  - either $c \prec d$
  - or ($c \sim d$ and uRv), such that R^* is serial:

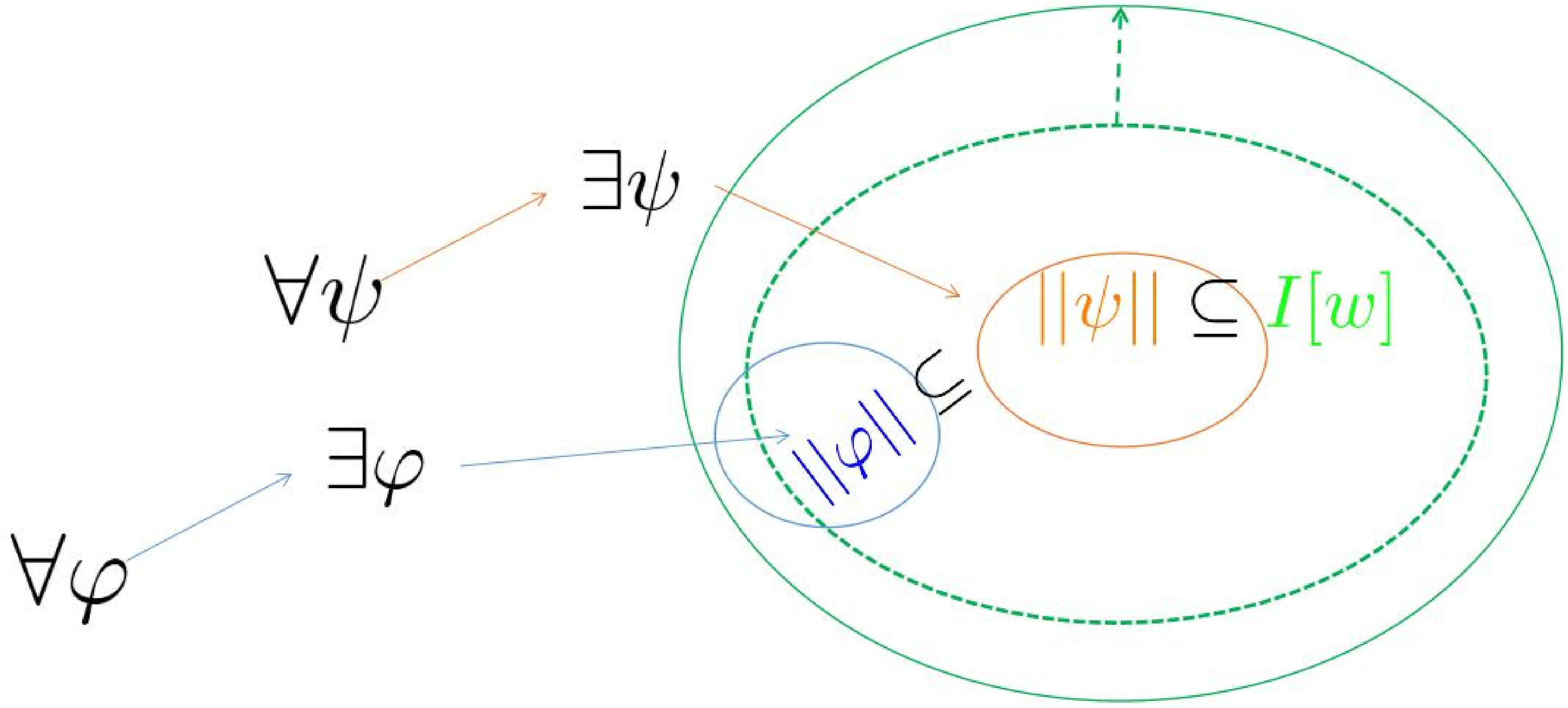
$$\forall(u, c)\exists(v, d) \text{ s.t. either } c \prec d \text{ or } (c \sim d \,\&\, uRv);$$

- $(u, c) \in V^*(p)$ iff $u \in V(p)$.

We need an additional axiom $c \rightarrow \bigvee_{d > c} Ed \vee \bigvee_{d \sim c} \neg O \neg d$ to characterize the seriality in the lexicographic-updated model.

“You are recovered well.”

- φ is “Take off your heart-rate monitor”
- ψ is “Take part in physical exercise at school”



Reduction Axioms of Lexicographic Update

- $[\Gamma]p \leftrightarrow \bigwedge_{c\in C}(\bigwedge c \rightarrow p)$
- $[\Gamma]\varphi \wedge \psi \leftrightarrow [\Gamma]\varphi \wedge [\Gamma]\psi$
- $[\Gamma]\neg\varphi \leftrightarrow \bigwedge_{c\in C}(\bigwedge c \rightarrow \neg[\Gamma]\varphi)$
- $[\Gamma](\varphi \trianglelefteq \psi) \leftrightarrow \bigwedge_{c\in C} \bigwedge_{d\in C}[c \rightarrow \bigvee_{e\succ d} A(\varphi^d \rightarrow E\psi^e) \vee \bigvee_{e\sim d}(\varphi^d \trianglelefteq \psi^e)]$
- $[\Gamma]O\varphi \leftrightarrow \bigwedge_{c\in C}[c \rightarrow \bigwedge_{d\succ c} A\varphi^d \wedge \bigwedge_{d\sim c} O\varphi^d]$
- $[\Gamma]P\varphi \leftrightarrow [(\bigwedge |\varphi| \leftrightarrow \bot) \rightarrow \bigwedge_{c\in C}[c \rightarrow \bigwedge_{d\in \max(|\varphi|)\cap\sim[c]} P \bigvee_{b\sim d} \varphi^b]]$

Concluding Remarks

- A sound and complete logic for defeasible permission is developed.
- Two dynamics of DEL updates for chaning permissions.
- Future work:

1. From “allow to be” to “allow to do”.
2. A multi-agent version “collectively allow to do”.
3. “Allow to know”

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